Z-plane specifications of

Analyse

Design in Digital Control

S-plane

Continuous

Analog control

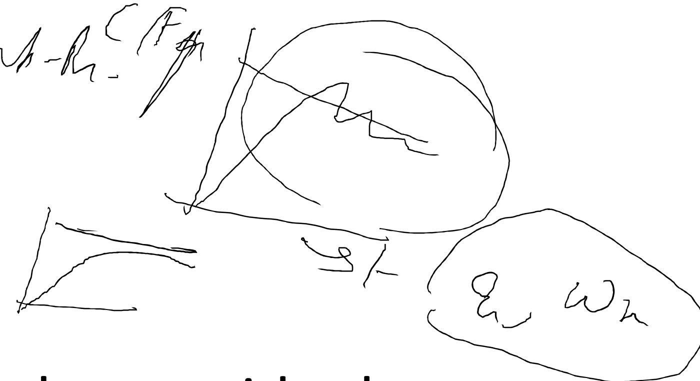
discretization

Digital

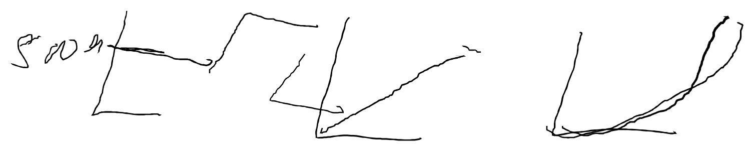
Time domain

Frequency domain

Unity feedback system



- Open loop transfer function $G_{h0}(z)G(z)$ having no poles outside the unit circle
- If the feedback is present then system it is desired to be an underdamped system.
- Transient response nature depends on system poles only
- Steady state response is dependent both on system and type of input
- Step(constant and easier)/Ramp(/Parabolic
- In practice we seldom find necessary to us a signal faster than parabolic



Steady State Accuracy

- Specification on steady state accuracy is often based on polynomial input of degree k $r(t) = \frac{t^k}{k!}\mu(t)$ k=0 step, k=1 ramp and k=2 parabolic

- Steady state error
- $e_{ss}^* = \lim_{k\to\infty}(r(k) - y(k))$
- Using Final value theorem
- $e_{ss}^* = \lim_{z\to 1}(z-1)E(z)$
- (z-1) E(z) has no poles on and outside the unit circle

- $\frac{Y(z)}{R(z)} = \frac{G_{h0}G(z)}{1+G_{h0}G(z)}$
- E(z)=R(z)-Y(z)=R(z)(1-$\frac{Y(z)}{R(z)}$)=$\frac{R(z)}{1+G_{h0}G(z)}$
- Forward path transfer function can be expressed as
- $G_{h0}(z)G(z) = \frac{K\prod_i(z-z_i)}{(z-1)^N\prod_j(z-p_j)}; p_j \neq 1, z_i \neq 1$
- In $(z-1)^N$ N indicates the number of poles on unit circle and it dominates the steady state error.

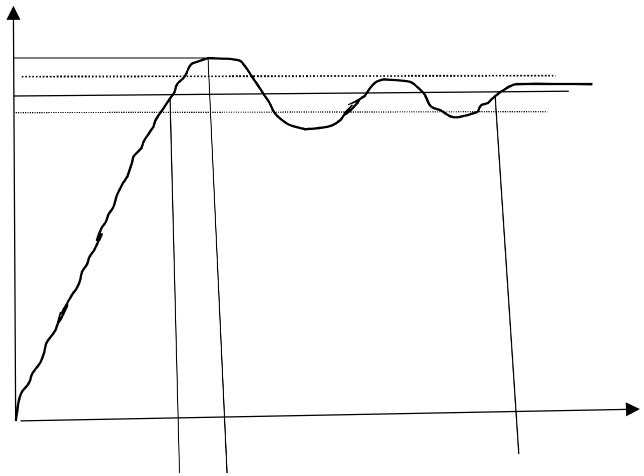
Unit Step Input	Ramp Input	Parabolic Input
$R(z) = \frac{z}{z-1}$	$R(z) = \frac{Tz}{(z-1)^2}$	$R(z) = \frac{T^2z(z+1)}{2(z-1)^3}$

Type of input	*Steady-state error*		
	Type-0 system	Type-1 system	Type-2 system
Unit step	$\frac{1}{1+K_p}$	0	0
Unit ramp	∞	$\frac{1}{K_v}$	0
Unit parabolic	∞	∞	$\frac{1}{K_a}$

$K_p = \lim_{z \to 1} G_{h0}G(z)$; $K_v = \frac{1}{T} \lim_{z \to 1} [(z-1)\, G_{h0}G(z)]$; $K_a = \frac{1}{T^2} \lim_{z \to 1} [(z-1)^2 G_{h0}G(z)]$

Transient Accuracy

- Typical unit step response of a digital control system is



Specifications in terms of root location in the z-plane

- transient response specifications are obtained in terms of characteristic roots in the s-plane and then using e^{Ts} to map the s-plane characteristic roots in the z-plane
- $\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2}$
- $t_r(0\%\ to\ 100\%) = \frac{\pi-\cos^{-1}\xi}{\omega_n\sqrt{1-\xi^2}}$
- $t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$
- $M_p = \exp(-{}^{\pi\xi}/_{\sqrt{1-\xi^2}})$
- $t_s(2\%\ tolerance\ band) = \frac{4}{\xi\omega_n}$
- Greater the magnitude of ω_n when ξ held constant speed of system is fast

- Value of ω_n is limited by measurement noise , system with large ω_n will have large bandwidth and will therefore allow the high frequency noise signals to affect the performance.

Now convert the specifications on ω_n and ξ to the characteristic root location in z-plane

s plane poles

$$s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

In z-plane

$$z_{1,2} = e^{-\xi\omega_n T} e^{\pm j\omega_n T\sqrt{1-\xi^2}} = re^{\pm j\theta}$$

Page No:
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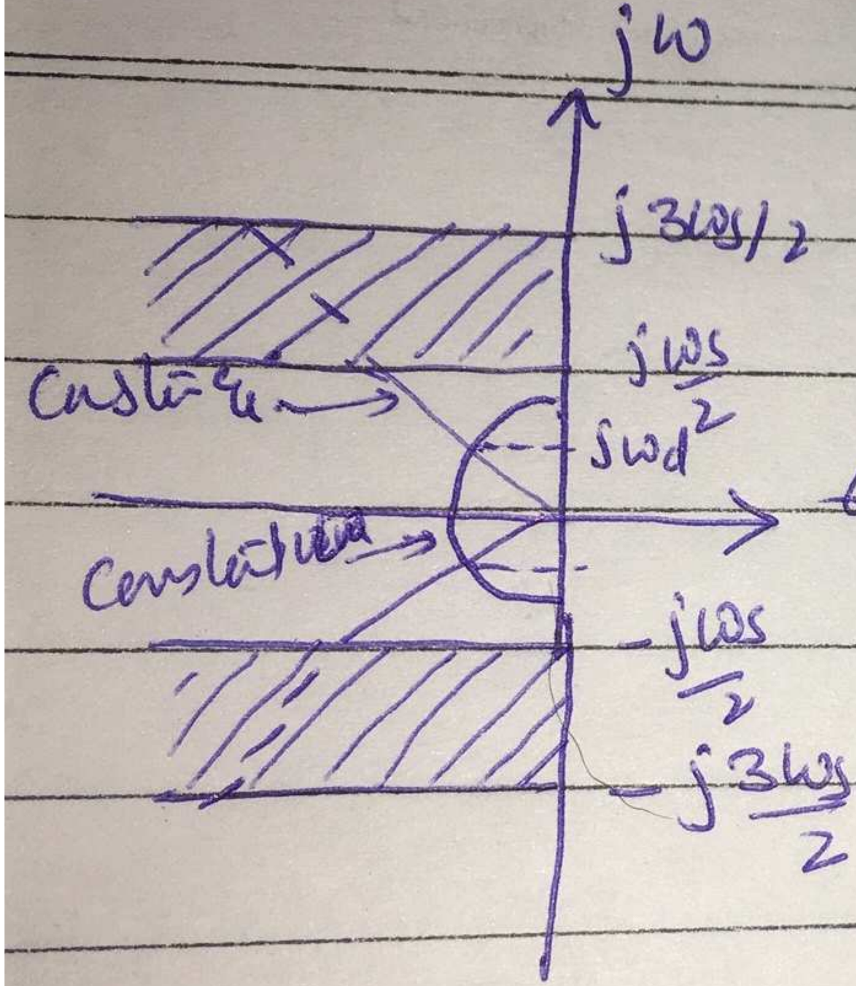
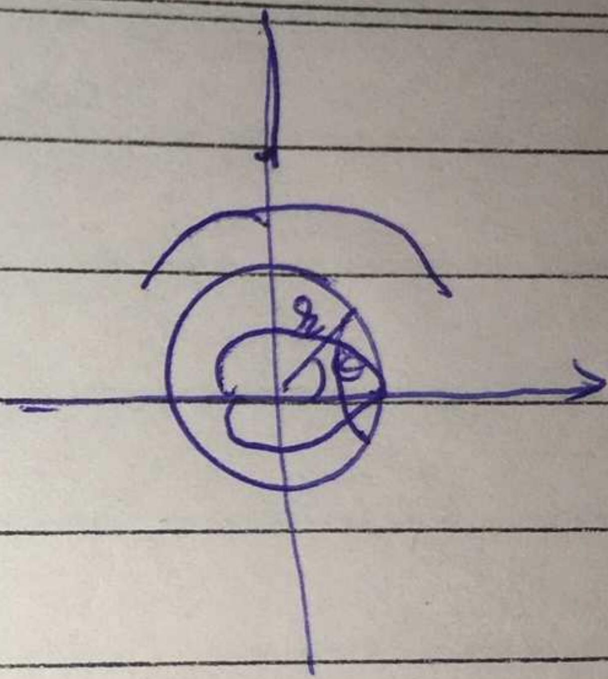


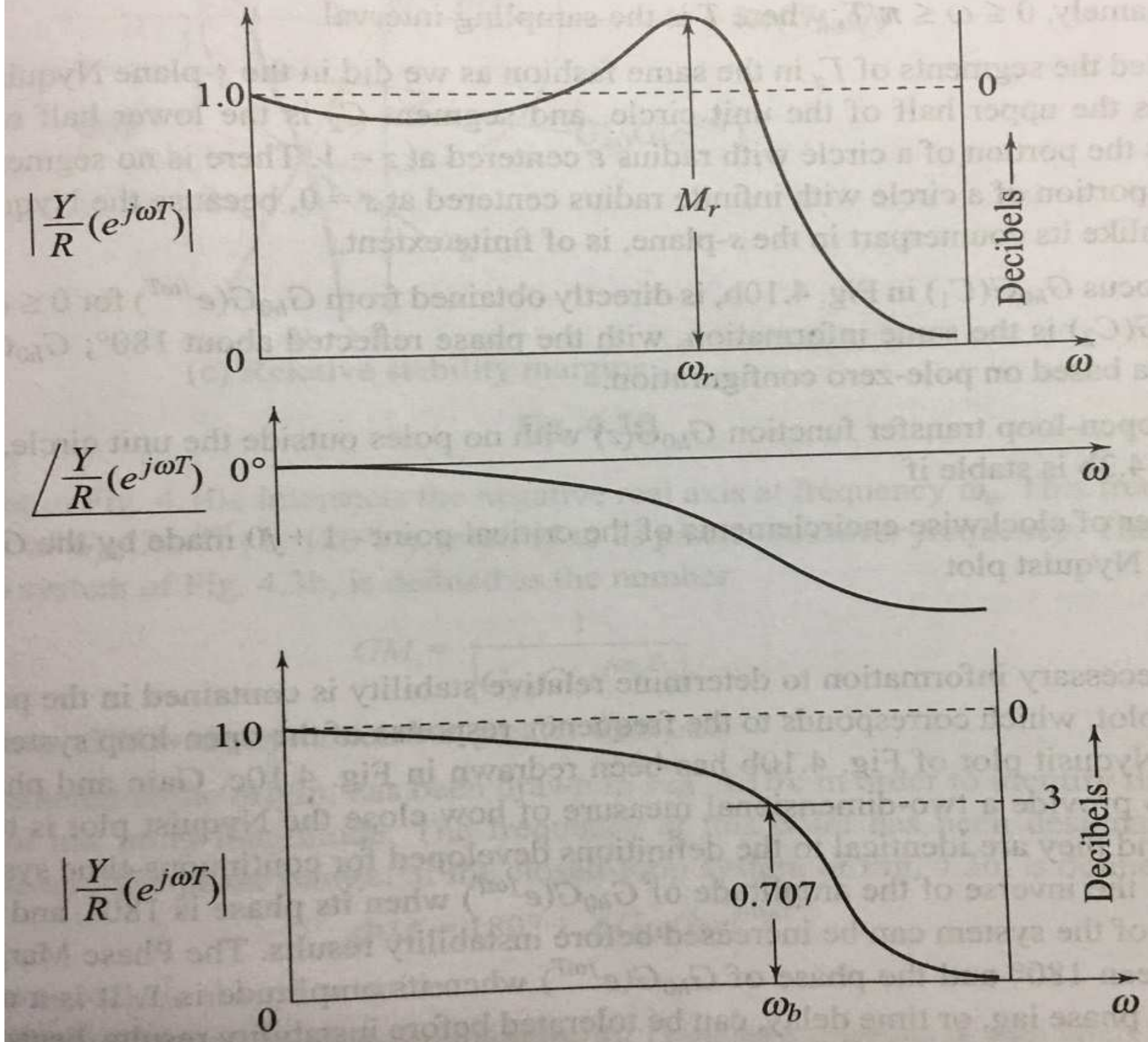


- Assume that low pass analog filtering characteristics of the continuous time plant and the ZOH device attenuate the response due to the poles in the complementary strips. Therefore only the poles in the primary strip are considered.

- In the z-plane CLP must lie on the constant ξ spiral to satisfy overshoot requirement and on $\omega_n$curve to satisfy speed of response.
- Intersection of two curves provide the preferred pole location
- For higher order systems addition of pole or a zero to a given system has only a small effect if the added singularities are in range 0 to -1
- Zero moving towards +1 increases overshoot, pole near +1 increases rise time

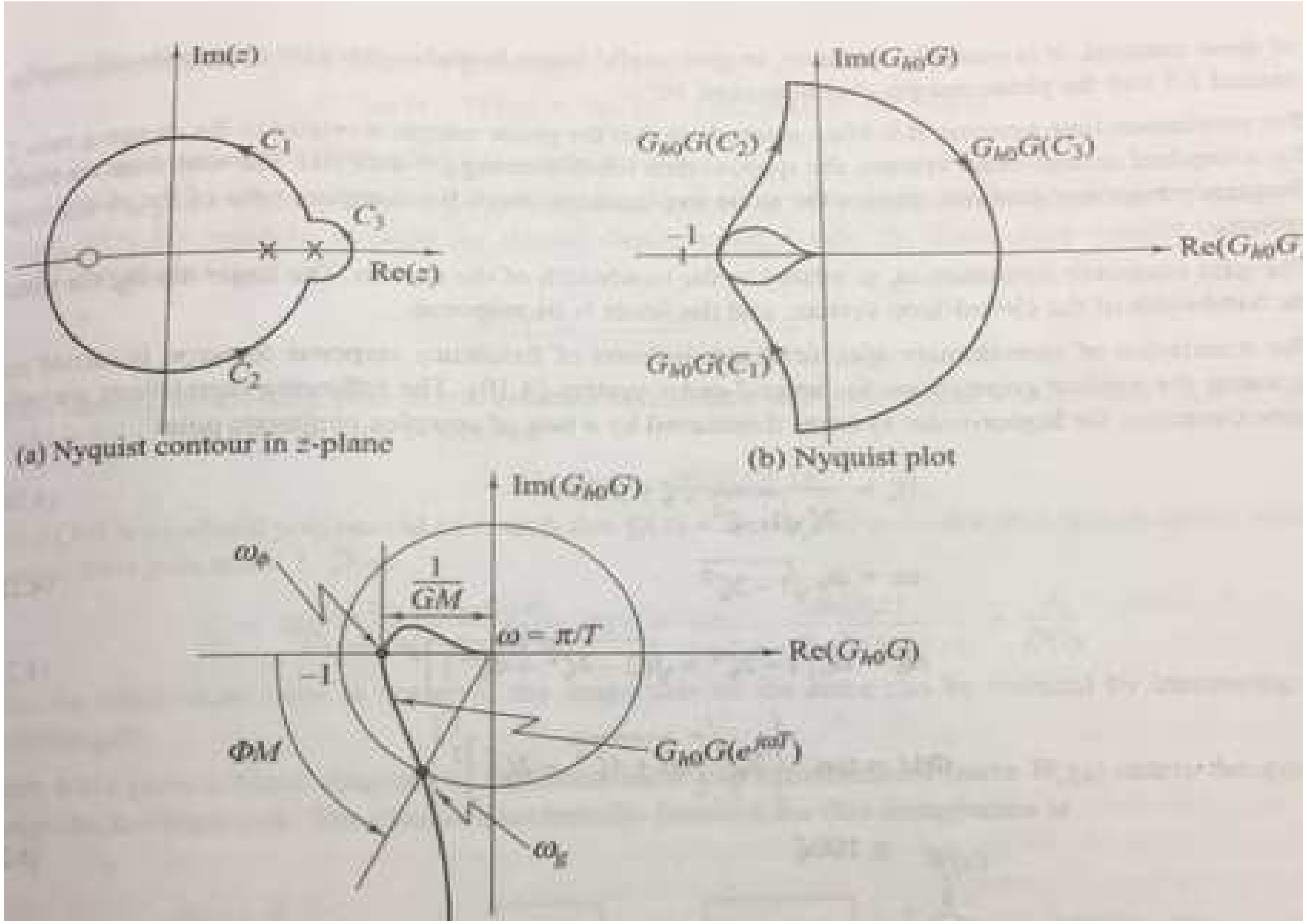
Specifications in terms of Frequency domain

- $\frac{Y(z)}{R(z)} = \frac{G_{h0}G(z)}{1+G_{h0}G(z)}$
- Z=$e^{j\omega T}$
- ω varies from -ω_s /2 to ω_s /2
- Mapping the unit circle plane onto $G_{h0}G(e^{j\omega T})$ plane
- Unit circle is symmetrical about real axis , $G_{h0}G(e^{j\omega T})$ awill also be symmetrical
- Need to plot only 0 to ω_s /2



1.0
0
Decibels
M_r
ω_r
ω
$\left|\frac{Y}{R}(e^{j\omega T})\right|$
$\angle\frac{Y}{R}(e^{j\omega T})$
0°
1.0
0
−3
Decibels
0.707
ω_b
0
ω

Nyquist Stability criterion in the z-plane



$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}; \zeta \leq 0.707$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

$$\omega_b = \omega_n\left[1-2\zeta^2+\sqrt{(2-4\zeta^2+4\zeta^4)}\right]^{\frac{1}{2}}$$

$$\Phi M = \tan^{-1}\left\{2\zeta\Big/\left[\sqrt{1+4\zeta^4}-2\zeta^2\right]^{\frac{1}{2}}\right\}$$

$$\cong 100\zeta$$