

Tutorial Sheet 1

1. Find the z-transforms of

a) $X(s) = \frac{1}{s(s+1)}$

b) $x(t) = e^{-at}$

c) $x(t) = A \cos(\omega t + \varphi)$

d) $x(t) = e^{-at} \cos \omega t$

2. Find the z-transforms using theorem

$$x(t) = e^{-at} \sin \omega t$$

3. Find the Z-transform of the following function.

A- $\mathbf{F}(s) = \frac{4}{s^2(s+2)}, \mathbf{F}(s) = \frac{2}{s^2+s+2}, \mathbf{F}(s) = \frac{2(s+1)}{s(s+2)}, \mathbf{F}(s) = \frac{10}{s(s^2+s+2)}$

B- $\mathbf{F}(k) = 0.1^k U_s(k) + 0.5k(0.1^k) U_s(k)$

4. Solve the following difference equation using z-transform

a) $y(k+2) + 3y(k+1) + 2y(k) = r(k); y(-1) = -0.5, y(-2) = 0.75$

b) $2y(k) - 2y(k-1) + y(k-2) = r(k) \quad y(k) = 0 \text{ for } k < 0 \text{ and}$

$$r(k) = \begin{cases} 1; & k = 0, 1, 2, \dots \\ 0; & k < 0 \end{cases}$$

c) $y(k) - 3y(k-1) + 2y(k-2) = r(k)$ where

$$r(k) = \begin{cases} 1 \text{ for } k = 0, 1 \\ 0 \text{ for } k \geq 2 \end{cases}; \quad y(-2) = y(-1) = 0$$

Will the final value theorem give the correct value of $y(k)$ as $k \rightarrow \infty$? Why?

5. For the transfer function models and inputs given below, find the response as a function of k

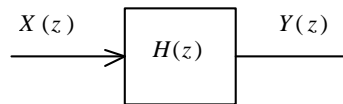
a) $G(z) = \frac{Y(z)}{R(z)} = \frac{2z-3}{(z-0.5)(z+0.3)} \quad r(k) = \begin{cases} 1, & k \text{ even} \\ 0, & k \text{ odd} \end{cases}$

b) $G(z) = \frac{Y(z)}{R(z)} = \frac{-6z+1}{(z-0.5+j0.25)(z-0.5-j0.25)} \quad r(k) = \begin{cases} 0, & k < 0 \\ 1, & k = 0, 1, 2, 3, \dots \end{cases}$

c) $G(z) = \frac{Y(z)}{R(z)} = \frac{1}{(z-0.5)(z+0.3)} \quad r(k) = \begin{cases} 1, & k \text{ even} \\ 0, & k \text{ odd} \end{cases}$

d) $G(z) = \frac{Y(z)}{R(z)} = \frac{1}{(z-0.5)^2(z-0.1)} \quad r(k) = \begin{cases} 0, & k < 0 \\ 1, & k = 0, 1, 2, 3, \dots \end{cases}$

6. Write down an expression in the z-domain for $Y(z)$ in terms of $X(z)$ and $H(z)$.



7. Write down an expression for $X(z)$ in terms of $x(n)$.
8. Write down an expression for $y(n)$ in terms of $x(n)$ and $h(n)$.
9. Show that your expression for $y(n)$ in question 3 is related to your expression for $Y(z)$ in question 1 by the z -transform.
10. Find the inverse z -transform of the system using partial fraction method and power series method

a) $X(z) = \frac{7z^2+5z}{z^2+2z+3}$

b) $X(z) = \frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$